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Elaboration and Diffusion of Useful Knowledge in the Long Run: The Case of European Practical Arithmetic (13th-16th Centuries)

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Elaboration and Diffusion of Useful Knowledge in the Long Run: The Case of European Practical Arithmetic (13th-16th Centuries)

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Abstract: The paper discusses the diffusion of Hindu-Arabic numerals in European commercial practices as a case study in the elaboration and transmission of useful knowledge in the pre-modern period. The reconstruction is based on a new dataset recording detailed information on over 1290 practical arithmetic manuals written from the late 13th century to 1600. Qualitative analysis and GIS visualisation tools make it possible to reconstruct a continuous spread of practical arithmetic linking the 'commercial revolution' to the European 'little divergence'. On the one hand, this discussion provides new evidence on the role of useful knowledge in the pre-modern period. On the other hand, it allows to raise new hypotheses on the channels through which useful knowledge spread, and on the dynamics of its economic impact.

Keywords: Commercial revolution, little divergence, human capital, useful knowledge, technical change

JEL codes: J24, N00, N13, N33, O30, O40

1. Introduction

As part of the debate on the 'great divergence', a growing stream of literature has stressed the role of culture and of 'useful knowledge' in the advent of modern economic growth (Pomeranz 2000; Mokyr 2005; 2011; 2017; Clark 2007). In parallel, a recent debate on the long-run determinants of the Industrial Revolution has brought forward the idea of a 'little divergence' of the economies of the Low Countries and England in the pre-modern period. This literature shows that these northern European areas experienced a persistent growth in terms of real wages, GDP per capita, urbanisation and agricultural productivity since the late Middle Ages, setting them on a trajectory that culminated with the Industrial Revolution (van Zanden 1999; Allen 2000; 2001; van Zanden and van Leeuwen 2012; Álvarez-Nogal and Escosura 2013; Broadberry *et al.* 2015). A debate ensued investigating the determinants of economic growth in the pre-modern period. A number of

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hypotheses emerged, with scholars first highlighting a relevant role of structural factors, such as success in international trade (Allen 2003), institutional change (Acemoglu, Johnson and Robinson 2005), and, more recently, the role of human capital and of access to information (van Zanden 2009; Dittmar 2011; De Pleijt and van Zanden 2016).

This paper contributes to this literature by focussing on the long-run development of European useful knowledge starting from the 'commercial revolution of the 13th century'. An established tradition of studies argues that this was a turning point in European economic history (De Roover 1953; Spufford 1988; Goldthwaite 2009; Tognetti 2015).[1] Innovations such as the *compagnia* (or 'partnership') contract, double-entry bookkeeping, insurance contracts, and the bill of exchange revolutionised European business methods, leading to the first appearance of the European merchant-banker (De Roover 1944; Parker and Yamey 1994; Mueller 1997; Padgett and McLean 2006; Goldthwaite 2009; Ceccarelli 2012; Bolton and Guidi Bruscoli 2021). These techniques and organisational innovations were marginally improved in subsequent periods, but their fundamental principles remained unchanged for centuries. The first major breakthroughs in commercial practices, such as the joint stock company and the stock exchange, were introduced in the 17th-century Low Countries. Following a cumulative rather than a revolutionary trajectory, these subsequent innovations did not substitute the techniques of the commercial revolution. Double-entry bookkeeping is still employed by most organisations, and the bill of exchange remained in wide use until the 20th century (Chaudhury and Denzel 2008). Nevertheless, we know surprisingly little of how these innovations were developed, and of how they spread throughout the European continent.

The main reason for our limited knowledge lies in the scarce availability of historical sources. For example, while the first experimentations of the bill of exchange were probably carried out around the middle of the 13th century, the oldest known bill of exchange is dated to 1304 (Spufford 1986, p. 44; Cella 2009). Moreover, the survival of accounting material is skewed. While thousands of account books written in Florence before 1500 are extant, it seems that no banker's account book written in Venice in the same period survived in its entirety (Goldthwaite 2018, p. 248). This paper sheds light on these issues by focussing on the diffusion of practical arithmetic with Hindu-Arabic numerals (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) among European practitioners. By providing a mathematical tool which allowed to better handle rational numbers, Hindu-Arabic numerals played a key role in the development and in the diffusion of the innovations of the commercial revolution.

The paper is based on a new dataset of practical arithmetic manuals dating from the late 13th to the end of the 16th century which was compiled by the author. This dataset gathers data on clearly identifiable sources, providing a higher level of detail and granularity than previous studies based

[1] The concept of the 'commercial revolution' has also been used to identify a series of wider socio-economic phenomena occurring in this period (Lopez 1976); following De Roover's original definition, however, the concept of 'commercial revolution' used in this paper is limited to business methods.

on estimates of book production (Baten and van Zanden 2008; Buringh and van Zanden 2009). As this dataset includes both manuscript and printed sources, it makes possible to examine the role of the printing press in the transmission of useful knowledge. Moreover, the evidence provided allows to identify a continuous process of elaboration and transmission of useful knowledge linking the commercial revolution to the little divergence of England and the Low Countries. This process occurred in phases which are discussed in detail. Furthermore, the paper raises observations on how useful knowledge spread, stressing the role of personal exchanges as key channels for its transmission.

The rest of the paper is organized as follows. Section 2 introduces the dataset and the methods used for its compilation. Section 3 discusses the European adoption of Hindu-Arabic numerals in the context of the commercial revolution of the 13th century. Section 4 discusses the diffusion of practical arithmetic in Italy in both manuscript form and in print. Section 5 provides an overview of the diffusion of practical arithmetic in western Europe, together with general considerations on the evidence provided. Section 6 wraps up and concludes.

2. Sources, data, and methods

The evidence presented in this paper is based on a new dataset compiled by the author which reconstructs the European tradition of practical arithmetic manuals – i.e. of texts which were used to teach and to learn practical arithmetic. While a number of practical arithmetic manuals have been studied by historians of mathematics, the entire tradition of European practical arithmetic has never been reconstructed as a coherent and continuous phenomenon. Practical arithmetic manuals provide particularly insightful sources to reconstruct the development of the practical knowledge of the commercial revolution for three reasons. First, Hindu-Arabic numerals facilitated commercial practices, as they made it possible to manipulate any rational number, as is discussed in Section 3. Second, practical arithmetic provided a foundational training for the merchants of the commercial revolution, and – contrary to other forms of commercial knowledge that were transmitted informally – its transmission occurred in dedicated institutional contexts. This also implies that these manuals survived in a less scattered way than evidence on informally transmitted techniques. Third, abacus manuals had a distinctly public circulation, and reached a considerably wide subset of Italian urban populations.

The database of practical arithmetic manuals is based both on primary and secondary sources. Catalogues are the main secondary source. The database records all the evidence provided by Van Egmond's *Practical Mathematics in the Italian Renaissance*, which remains the most comprehensive catalogue for the tradition of Italian abacus manuals (Van Egmond 1980). This corpus records 288 manuscript and 153 printed practical arithmetic manuals written before 1601. Whenever possible, these texts have been inspected directly, either by examining original copies or through digital re-

productions. This made it possible to manually record the contents of both manuscript and printed manuals, as Van Egmond's catalogue only provides summaries of contents for manuscript texts. The evidence from this catalogue was updated with the results of subsequent research in the field.[2] Moreover, this evidence was complemented with archival research in Florence (Biblioteca Nazionale Centrale, Biblioteca Riccardiana, Biblioteca Mediceo-Laurenziana) and Bologna (Biblioteca Universitaria), which provided first-hand familiarity with primary sources, and allowed to identify a number of new works.

The evidence on the European tradition of practical arithmetic is based on a variety of sources. The fundamental secondary source is the catalogue by Smith, which is based on a monumental effort of archival research (Smith 1908). As this catalogue includes all kinds of mathematical works written before 1601, its contents were selected to include only works that use Hindu-Arabic numerals and that belong to the tradition of practical arithmetic (i.e. works that either include practical applications of mathematics, are addressed to practitioners, or could be used for commercial training). This implies that theoretical works have been excluded, together with printed editions of mathematical texts from classical antiquity.

This evidence was updated by consulting relevant online repositories, such as the Incunabula Short Title Catalogue (ISTC), the Universal Short Title Catalogue (USTC), the Biblioteca Virtual Miguel de Cervantes (Cervantes Virtual), the Münchener DigitalisierungsZentrum of the Bayerische Staatsbibliothek, the online catalogue of the Bibliothèque Nationale de France (Gallica), and the English Short Title Catalogue (ESTC). These resources, which were not available to either Smith or Van Egmond, made it possible to broaden the evidence available in their catalogues, for example by identifying new authors, new works by the same author, and previously unrecorded editions.

The selection criteria used for Smith's catalogue and the consultation of online repositories were applied to all other relevant catalogues. Among these are Smith's own update to his catalogue (Smith 1939), Navarro Brotons' valuable catalogue on Iberian works (Navarro Brotons 2000), and the *Ars mercatoria*, which made it possible to strengthen the evidence on central and northern European printed works (Hoock, Jeannin and Kaiser 1991). Also in this case, the evidence provided by these catalogues was complemented by the findings of specialised studies.[3] Moreover, the evidence on the European tradition was complemented by archival research, carried out in the Bibliothèque Nationale de France (Paris), the British Library (London), and the Cambridge University Library (UK).

The final dataset includes 1.290 works written before 1600. Of these, 349 are manuscript documents and 941 are printed works. In comparison with Van Egmond's catalogue, this dataset provides over 20% more manuscript sources and over five times as many printed ones. The dataset records docu-

[2] Such as Ulivi 2002a; Franci 2003; 2015; Long *et al.* 2009; Bocchi 2017.

[3] Single papers are quoted in the relevant passages of this text.

Table 1. *Number of practical arithmetic manuals for area of publication and sub-period*

	1250-1300	1301-1350	1351-1400	1401-1450	1451-1500	1501-1550	1551-1600
Austria	0	0	0	0	3	5	1
Bohemia	0	0	0	0	0	1	8
Denmark	0	0	0	0	0	0	5
England	0	0	0	0	0	9	46
France	0	3	0	2	11	37	92
Germany	0	0	0	1	12	158	222
Hungary	0	0	0	0	0	0	3
Italy	2	19	24	60	133	119	126
Low Countries	0	0	0	0	0	12	63
Poland	0	0	0	0	0	6	13
Portugal	0	0	0	0	0	4	4
Spain	0	0	3	0	4	10	44
Switzerland	0	0	0	0	0	5	18
Total	2	22	27	63	163	366	645

Source: Database described in Section 2.

ments written in more than 130 cities by over 340 authors.[4] While the database records manuscript works at the copy level, printed works are recorded at the edition level. Even if the dataset is based on the best evidence available for the period from the 13th to the 16th century, this represents a limitation of the evidence recorded. While this paper is addressed to economic historians, this dataset is also relevant for the history of science and other fields of historical research (Danna 2021; 2023). In this paper, the evidence recorded is analysed with qualitative methods, GIS visualisation tools, and descriptive statistics. An econometric analysis of the measurable impact of this tradition is currently in progress (Danna, Iori and Mina 2022). Table 1 summarises the number of manuals for areas and sub-periods.

3. Hindu-Arabic numerals, the commercial revolution, and useful knowledge

Before the introduction of Hindu-Arabic numerals, Europeans resorted to a variety of reckoning devices, the most common of which was the reckoning table, also referred to as 'counter abacus' (Barnard 1916; Day 2013). The key mathematical innovation brought to the Latin west by Arabic arithmetic was the positional numeral system. Probably first developed in 5th-century India, the positional numeral system has three important characteristics: it is symbolic, it is based on the principle of place value, and it introduced the symbol of zero (Ifrah 2000, p. 679). Thanks to these features, the positional numeral system makes it possible to calculate algorithmically with the ten digits, providing at the same time a reckoning device (a means for calculation) and a recording device (a means to write values in a fixed form).

[4] The number is an estimation because several manuals were written by anonymous authors.